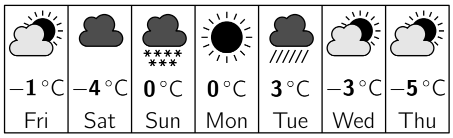
Year 11 to 13 (ENGLISH VERSION)

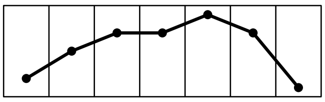
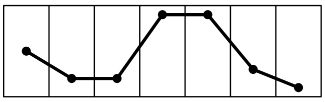
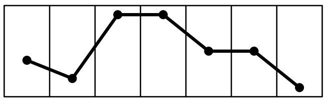
Thursday, 18th March 2021 Time allowed: 75 minutes

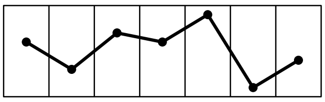
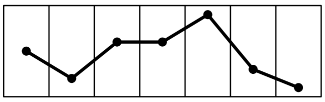
1. For each question exactly one of the 5 options is correct.
2. Each participant is given 30 points at the beginning. For each correct answer 3, 4 or 5 points are added. No answer means 0 points are added. If a wrong answer is given, one quarter of the points is subtracted, i. e. 0.75 points, 1 point or 1.25 points, respectively. At the end, the maximum number of points is 150, the minimum is 0.
3. Calculators and other electronic devices are not allowed.

3 point problems

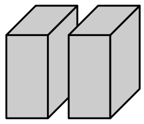
A1 The weather app on Magdalena's mobile phone shows the expected maximum temperatures for the next seven days (see figure). What does the corresponding graph look like?



(**A**)  (**B**)  (**C**) 

(**D**)  (**E**) 

A2 A cube whose edges have length 1 m is halved in such a way that two identical cuboids arise. What is the surface area of such a cuboid?



(**A**) $3\,\text{m}^2$ (**B**) $4\,\text{m}^2$ (**C**) $5\,\text{m}^2$ (**D**) $6\,\text{m}^2$ (**E**) $7\,\text{m}^2$

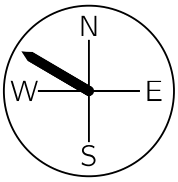
A3 How many natural numbers are greater than $20-\sqrt{21}$ and less than $20+\sqrt{21}$?

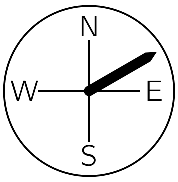
(**A**) 9 (**B**) 10 (**C**) 11 (**D**) 12 (**E**) 13

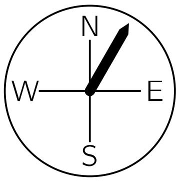
A4 In August, many shooting stars can be observed at night. Last year, Jesko saw 100 shooting stars within one hour. That means on average he saw one shooting star every

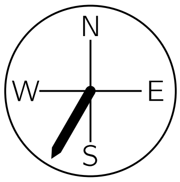
(**A**) 28 seconds. (**B**) 30 seconds. (**C**) 36 seconds. (**D**) 40 seconds. (**E**) 42 seconds.

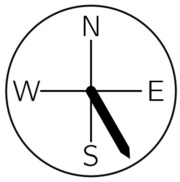
A5 Last night's storm bent the flagpole in front of our school. If you look at the pole from the north or from the east, it leans to the right both times. One of the pictures shows in which direction the pole is bent. Which one?

(**A**) 

(**B**) 

(**C**) 

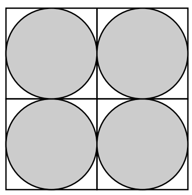
(**D**) 

(**E**) 

A6 Which of the following numbers is the largest?

(**A**) 0.815^4 (**B**) 0.815^2 (**C**) 0.815 (**D**) $\sqrt{0.815}$ (**E**) $\sqrt[4]{0.815}$

A7 A large square is divided into four smaller squares (see figure). A grey circle is inscribed into each of the small squares. What proportion of the area of the large square is grey?



(**A**) $\frac{4}{5}$ (**B**) $\frac{3\pi}{16}$ (**C**) $\frac{3}{\pi}$ (**D**) $\frac{3}{4}$ (**E**) $\frac{\pi}{4}$

A8 How many two-digit natural numbers are divisible by 3 and have only odd digits?

(**A**) 2 (**B**) 3 (**C**) 4 (**D**) 8 (**E**) 12

A9 The points $A(p\,|\,q)$, $B(3p\,|\,q)$ and $C(2p\,|\,3q)$, where $p > 0$ and $q > 0$, are plotted into a coordinate system. What is the area of the triangle ABC?

(**A**) $\frac{1}{2}pq$ (**B**) $2pq$ (**C**) $3pq$ (**D**) $4pq$ (**E**) $\frac{9}{2}pq$

A10 Amanda thinks of a rational number between 2 and 4, Justus of one between 7 and 8 and Theresa of one between 5 and 6. What is certainly true about the sum S of these three numbers?

(**A**) $S \leq 17$ (**B**) $S \geq 16$ or $S \leq 14$ (**C**) $S \geq 14$ and $S \leq 18$
(**D**) $S \geq 15$ and $S \leq 22$ (**E**) $S \geq 17$ or $S \leq 15$

4 point problems

B1 Two pallets with a total of 60 beverage crates were delivered to a supermarket. Some crates were immediately unloaded from one pallet. Both before and after unloading, there were 1.5 times as many crates on one pallet as on the other. How many crates were unloaded immediately?

(**A**) 10 (**B**) 15 (**C**) 16 (**D**) 20 (**E**) 24

B2 What proportion of the positive divisors of $5! = 1 \times 2 \times 3 \times 4 \times 5$ is odd?

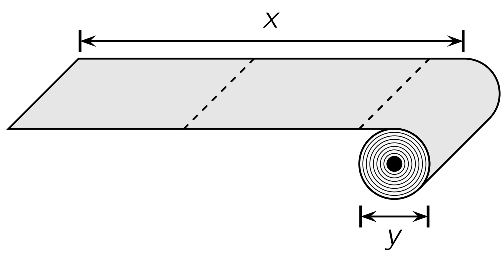
(**A**) $\frac{1}{2}$ (**B**) $\frac{1}{3}$ (**C**) $\frac{1}{4}$ (**D**) $\frac{1}{5}$ (**E**) $\frac{1}{6}$

B3 A rectangular sheet of paper has length x cm and width y cm, where $x > y$. If you glue the two long sides or the two short sides together, you get the shell of a taller or a flatter cylinder. What is the ratio of the volume of the taller cylinder to the volume of the flatter cylinder?

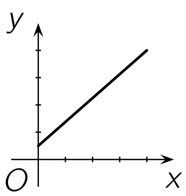
(**A**) $1 : y^2$ (**B**) $y^2 : x^2$ (**C**) $x : y^2$ (**D**) $y : x$ (**E**) $x : 1$

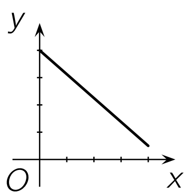
B4 How many three-digit natural numbers have the property that by reversing the order of the digits the number increases by 99?

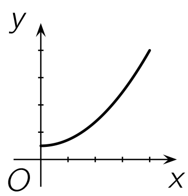
(**A**) 56 (**B**) 64 (**C**) 72 (**D**) 80 (**E**) 81

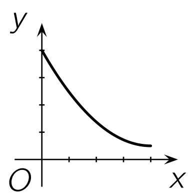


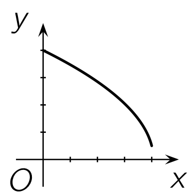
B5 When a roll of kitchen paper is unrolled evenly, the diameter y of the roll becomes smaller and smaller over time, while the total length x of the already unrolled part becomes larger and larger. Which graph represents this process?

(**A**) 

(**B**) 

(**C**) 

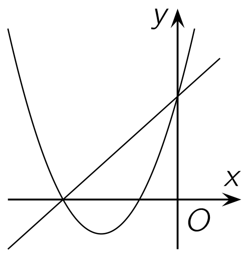
(**D**) 

(**E**) 

B6 The natural numbers from 1 to 100 are written next to each other in some order. Now all sums of three adjacent numbers are calculated. How many of these sums can be odd *at most*?

(**A**) 97 (**B**) 96 (**C**) 95 (**D**) 94 (**E**) 93

B7 The figure shows the parabola with the equation $y = ax^2 + bx + c$ for three different real numbers a, b, c and a straight line. Which of the following equations could possibly describe this line?



(**A**) $y = bx + c$ (**B**) $y = cx + b$ (**C**) $y = ax + b$
(**D**) $y = ax + c$ (**E**) $y = cx + a$

B8 Ahmed wants to write the numbers from 1 to 25 in the 5×5 field so that the five numbers in each row and the five numbers in each column have the same sum. Ahmed has already written some of the numbers. What number does Ahmed have to write in the box with the question mark?

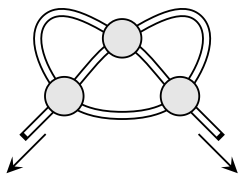
	24		5	
22		17		4
	11		15	
25		9		1
	2		?	

(**A**) 7 (**B**) 12 (**C**) 14 (**D**) 19 (**E**) 21

B9 The function f is defined for all real numbers. It is $f(1) = 2$, and $f(x+y) = f(x) \times f(y)$ is true for all real numbers x and y. Then $\dfrac{f(2021)}{f(2020)} =$

(**A**) 2 (**B**) $\dfrac{2020}{2021}$ (**C**) 2022 (**D**) $\dfrac{2022}{2021}$ (**E**) 4044

B10 Isa places a piece of rope on the table as shown. At each of the three crossings she randomly decides whether the crossing looks like this  or like this , each with a probability of $\frac{1}{2}$. What is the probability of the rope becoming knotted when Isa pulls on the two ends of the rope?



(**A**) $\frac{1}{2}$ (**B**) $\frac{1}{4}$ (**C**) $\frac{1}{8}$ (**D**) $\frac{3}{4}$ (**E**) $\frac{3}{8}$

5 point problems

C1 Our basketball team won their last game. The 7 players scored 1, 2, 7, 9, 10, 15 and 19 points. The three tallest players altogether scored twice as many points as the three shortest players altogether. How many points did the fourth tallest player score?

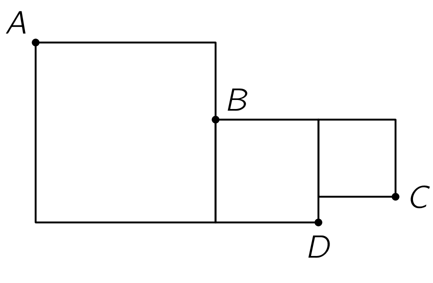
(**A**) 7 (**B**) 9 (**C**) 10 (**D**) 15 (**E**) 19

C2 Jesse and Liv play Battleship in a slightly different way. Jesse places a ship on one of the four squares (see figure). Liv tries to find out the location of the ship. To do this, she asks for one of the four squares. If the ship is there, the game is over. If not, Jesse moves the ship one square to the right or to the left and Liv can ask again for a square. Which of the following sequences of questions allows Liv to find the ship for sure?

1	2	3	4

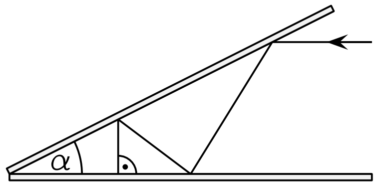
(**A**) 1,2,3,4,1,2,3,4 (**B**) 1,2,3,4,3,2,1 (**C**) 1,3,1,3,1,3 (**D**) 1,4,4,1,1 (**E**) 2,3,3,2

C3 Three squares are shown on the right. The points A, B and C lie on a straight line. The area of the left square is $49\,\text{cm}^2$ and the area of the middle square is $16\,\text{cm}^2$. What is the area of the triangle ADC?



(**A**) $24.5\,\text{cm}^2$ (**B**) $22\,\text{cm}^2$ (**C**) $18.5\,\text{cm}^2$ (**D**) $16\,\text{cm}^2$ (**E**) $14.5\,\text{cm}^2$

C4 Two mirrors are joined as shown and enclose an angle of size α. A ray of light is incident parallel to one of the mirrors and is reflected back along the same path after the fourth reflection. What is α? (*figure not to scale*)

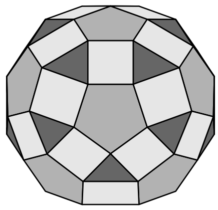


(**A**) 10° (**B**) 15° (**C**) 22.5° (**D**) 30° (**E**) 45°

C5 Finn runs at a constant speed following his friend Luisa, who is just crossing a bridge, also at a constant speed. If Luisa were to turn around as soon as Finn enters the bridge, she would meet Finn in the middle of the bridge after 150 metres. But Luisa doesn't notice him, and so Finn only catches up with her at the end of the bridge. How long is the bridge?

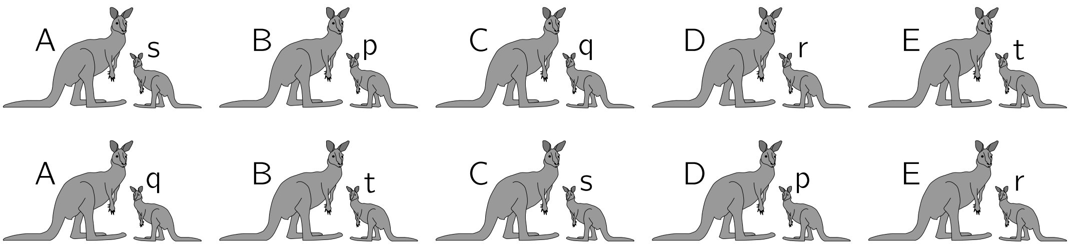
(**A**) 525 m (**B**) 600 m (**C**) 750 m (**D**) 775 m (**E**) 900 m

C6 The picture shows a polyhedron consisting only of equilateral triangles, squares and regular pentagons. Each pentagon is adjacent to five squares, each triangle is adjacent to three squares, and each square is adjacent to two triangles and two pentagons. In total there are 12 pentagons. How many triangles are there?



(**A**) 18 (**B**) 20 (**C**) 24 (**D**) 25 (**E**) 30

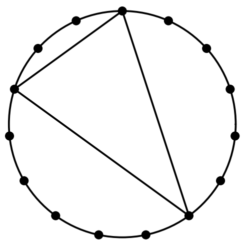
C7 Five young kangaroos p, q, r, s, t and their five mothers A, B, C, D, E stand in a row.



In the top row, exactly 2 of the young kangaroos stand with their own mothers. In the bottom row, exactly 3 of the young kangaroos stand with to their own mothers. Which kangaroo is the mother of p?

(**A**) A (**B**) B (**C**) C (**D**) D (**E**) E

C8 On a circle, 15 points are marked in such a way that neighbouring points always have the same distance. If you connect three of these points, you get a triangle. How many different, non-congruent triangles can be drawn in this way?

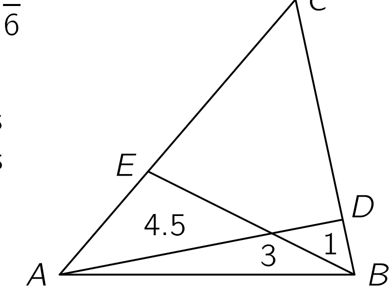


(**A**) 19 (**B**) 46 (**C**) 15 (**D**) 75 (**E**) 23

C9 Emma and Linda toss a coin. Each time tails comes up, Emma gets a point, with heads Linda gets a point. The game is won by the first player to have three points more than the other. Tails comes up on the first toss. What is the probability that Emma will win?

(**A**) $\frac{1}{2}$ (**B**) $\frac{2}{3}$ (**C**) $\frac{3}{4}$ (**D**) $\frac{4}{5}$ (**E**) $\frac{5}{6}$

C10 The triangle ABC is divided into four parts by the two line segments $\overline{AD}$ and $\overline{BE}$. The areas of the three small triangles are 1, 3 and 4.5, as indicated. What is the area of the triangle ABC?



(**A**) 18 (**B**) 20 (**C**) 21 (**D**) 22 (**E**) 24