Year 11 to 13 (ENGLISH VERSION)

Thursday, 15th March 2018

Time allowed: 75 minutes

1. For each question exactly one of the 5 options is correct.
2. Each participant is given 30 points at the beginning. For each correct answer 3, 4 or 5 points are added. No answer means 0 points are added. If a wrong answer is given, one quarter of the points is subtracted, i.e. 0.75 points, 1 point or 1.25 points, respectively. At the end, the maximum number of points is 150, the minimum is 0.
3. Calculators and other electronic devices are not allowed.

3 point problems

A1 When the 3rd of March in a particular year is a Saturday, so the 27th of March of that same year is a
(A) Monday. (B) Tuesday. (C) Wednesday. (D) Thursday. (E) Friday.

A2 Which of the following calculations gives the largest result?
(A) \( \frac{20 \times 18}{15 \times 3} \) (B) \( \frac{18 \times 15}{20 \times 3} \) (C) \( \frac{18 \times 3}{20 \times 15} \) (D) \( \frac{20 \times 15}{18 \times 3} \) (E) \( \frac{20 \times 3}{18 \times 15} \)

A3 For a brunch buffet, Mr. Snyder cooked 7 hard-boiled eggs. With his egg slicer, Mr. Snyder cuts some of them into 5 pieces. How many pieces – whole eggs and egg parts – could he obtain in total?
(A) 17 (B) 20 (C) 21 (D) 23 (E) 25

A4 When Monika locked the front door, she noticed that she was not wearing her glasses. Back inside, she rushed through each door in her flat exactly once before she finally found her glasses. In which room did Monika find her glasses?
(A) room 1 (B) room 2 (C) room 3 (D) room 4 (E) room 5

A5 Which of the following numbers is closest to the value of \( 0.435 \div 0.0821 \)?
(A) 0.2 (B) 0.5 (C) 5 (D) 20 (E) 50

A6 A disc is divided into two parts such that the ratio of the areas of the two parts is \( 2 : 7 \). What is the probability that a randomly chosen point of the disc belongs to the larger of the two parts?
(A) \( \frac{7}{9} \) (B) \( \frac{5}{6} \) (C) \( \frac{3}{5} \) (D) \( \frac{5}{9} \) (E) \( \frac{4}{7} \)

A7 Four congruent rhombuses and two squares are put together to make a regular octagon, as shown. What is the size of the smaller angle of each rhombus?
(A) 30° (B) 35° (C) 36° (D) 40° (E) 45°
A8 There are two types of badgers: Meles and Arctonyx. Arctonyx is found only in East and Southeast Asia. Which of the following statements can therefore be concluded?

(A) All badgers are found in Asia.  (B) In Asia, only Arctonyx is found.
(C) Meles is found in America.  (D) Meles is found only in Africa.
(E) Arctonyx is not found in Australia.

A9 In how many ways can the number 1001 be written as a sum of two prime numbers?

(A) 0  (B) 1  (C) 3  (D) 5  (E) 8

A10 Under a running water tap, Karoline fills a vase with water up to the top. The graph shows the height $h$ of the water in the vase as a function of the time $t$. Which of the following could be the shape of the vase that Karoline just filled?

(B) 4

4 point problems

B1 In a store, Kostas compares two smartphones that are currently for sale. The first one costs 90% and the second one 85% of its original price. With either smartphone, Kostas would save the same amount of money compared to the original price. The original price of the first smartphone was $p$ euros. What was the original price of the second smartphone?

(A) $\frac{2}{3}p$ euros  (B) $\frac{3}{2}p$ euros  (C) $\frac{17}{18}p$ euros  (D) $\frac{18}{17}p$ euros  (E) $p$ euros

B2 $|\sqrt{17} - 5| + |\sqrt{17} + 5| =$

(A) 10  (B) $2 \times \sqrt{17}$  (C) $\sqrt{34} + 10$  (D) $2 \times \sqrt{17} + 10$  (E) 0

B3 The centers of the faces of a cube are the vertices of an octahedron, as indicated in the diagram. The side length of the cube is 1. What is the volume of the octahedron?

(A) $\frac{1}{3}$  (B) $\frac{1}{4}$  (C) $\frac{1}{5}$  (D) $\frac{1}{6}$  (E) $\frac{1}{8}$

B4 Marion bought plants for her balcony: 3 pansies in 3 different colours and 6 primroses in 6 different colours. In a large bowl she wants to plant 2 pansies and 4 primroses. In how many ways can Marion choose them from the plants she bought?

(A) 24  (B) 30  (C) 45  (D) 60  (E) 96

B5 The prism shown in the diagram is formed by two triangles and three rectangles. The six vertices of this prism should be labelled with the numbers from 1 to 6 in such a way that for each rectangle the sum of the numbers at the four vertices is the same. One vertex is already labelled with 1. What is $x$?

(A) 2  (B) 3  (C) 4  (D) 5  (E) 6
B6. The faces of a cuboid have areas $A$, $B$ and $C$, as shown.
What is the volume of this cuboid?

- (A) $ABC$
- (B) $\sqrt{ABC}$
- (C) $\sqrt[3]{ABC}$
- (D) $\sqrt{AB + BC + CA}$
- (E) $\sqrt{A^3 + B^3 + C^3}$

B7. Two rectangles touch a straight line as shown (diagram not to scale).
What is the size of the angle marked $\alpha$?

- (A) $70^\circ$
- (B) $90^\circ$
- (C) $105^\circ$
- (D) $110^\circ$
- (E) $120^\circ$

B8. Which of the following numbers is not a divisor of $18^{2017} + 18^{2018}$?

- (A) $8$
- (B) $18$
- (C) $28$
- (D) $38$
- (E) $48$

B9. Before the cycle ball final, CBC Chain – SBC Spoke, five predictions were made:
1. The match will not end in a draw.
2. CBC Chain will score at least one goal.
3. CBC Chain will not lose.
4. Three goals will be scored in total.
5. CBC Chain will win.

After the match it turned out that exactly three of the predictions came true. What was the final score of the match?

- (A) $3 - 0$
- (B) $2 - 1$
- (C) $1 - 1$
- (D) $0 - 3$
- (E) $1 - 2$

B10. The vertices of a triangle have coordinates $(a \mid b)$, $(c \mid d)$ and $(e \mid f)$. The midpoints of the sides of this triangle have coordinates $(-2 \mid 1)$, $(2 \mid -1)$ and $(3 \mid 2)$. What is the value of $a + b + c + d + e + f$?

- (A) $2$
- (B) $2.5$
- (C) $3$
- (D) $5$
- (E) $10$

5 point problems

C1. Three of the five cards shown are given to Paula and the others to Dominic.
Paula multiplies the 3 values of her cards and Dominic multiplies the 2 values of his cards. It turns out that the sum of the two resulting products is a prime number. What is the sum of the values of Paula’s cards?

- (A) $12$
- (B) $13$
- (C) $15$
- (D) $17$
- (E) $18$

C2. Knowsnot and Doesnot have to get form $A$ from the joke authority. Each of the forms $A$, $B$, $C$, $D$ and $E$ are available at exactly one of the following five counters.

- Counter 1: Here you receive form $B$.
- Counter 2: Here you receive form $C$ or $E$.
- Counter 3: Here you receive form $D$.
- Counter 4: Here you receive form $A$, $C$ or $E$.
- Counter 5: Here you receive form $A$.

Knowsnot is confused when he reads the signs at the counters. Doesnot remembers what they were told by the receptionist: Exactly one statement on the signs is false, and the other four statements are true. At which counter will they receive form $A$?

- (A) at counter 1
- (B) at counter 2
- (C) at counter 3
- (D) at counter 4
- (E) at counter 5
C3 The quadratic equation $x^2 - x - 2018 = 0$ has two different roots $x_1$ and $x_2$. What is the value of $x_1^2 + x_2$?

(A) 2016    (B) 2017    (C) 2018    (D) 2019    (E) 2020

C4 From a piece of striped paper a regular pentagon is cut out. Then, this pentagon is rotated counterclockwise around its centre, at each step by $21^\circ$. On the right, the situation after the first step is shown. What will be seen when the pentagon fits back in the hole for the first time?

(A) (B) (C) (D) (E)

C5 The function $g$ satisfies $g(1) = \frac{1}{2}$ and $g(x + y) = g(x)g(y)$ for all integers $x$ and $y$. What is the value of $g(0) + g(1) + g(2) + g(3)$?

(A) $\frac{15}{8}$    (B) $\frac{3}{2}$    (C) $\frac{7}{4}$    (D) $\frac{13}{16}$    (E) $\frac{5}{2}$

C6 A billiard player practices at a rectangular billiard table that has a playing surface of $2.50 \times 1.30$ m. He places a ball in point $A$ at the front side $2.25$ m away from pocket $B$. Then, the ball hits the right side in point $C$ which is $90$ cm away from pocket $B$. After two reflections, the ball stops at the left side, as shown. How far is the ball now away from the midpoint of the left side?

(A) 2 cm    (B) 5 cm    (C) 9 cm    (D) 10 cm    (E) 12 cm

C7 The graph of the quadratic function $f(x) = x^2 + px + q$ intersects the $x$-axis and the $y$-axis in three different points. The circle through these three points intersects the graph of $f$ in a fourth point. What are the coordinates of this fourth point?

(A) $\left(-\frac{q}{p}, \frac{q^2}{p^2}\right)$    (B) $(p, q)$    (C) $(-p, q)$    (D) $(0, -q)$    (E) $(1, p + q + 1)$

C8 How many real solutions does the equation $|4^x - 3| - 2 = 1$ have?

(A) 2    (B) 3    (C) 4    (D) 5    (E) 6

C9 In a field trial for hunting dogs, long-haired dachshunds and wire-haired dachshunds take part. There are $40\%$ more long-haired dachshunds than wire-haired dachshunds. The two dachshunds for the first task are drawn now. The probability that a long-haired dachshund will compete against a wire-haired dachshund is exactly $50\%$. How many dachshunds in total take part in the field trial?

(A) 24    (B) 30    (C) 36    (D) 38    (E) 48

C10 Archimedes calculated $15! = 1 \times 2 \times 3 \times \ldots \times 15$ and wrote down the result. Unfortunately, the second digit and the tenth digit cannot be read anymore: $1\underline{0767436}000$. What are these two digits?

(A) 2 and 0    (B) 4 and 8    (C) 7 and 4    (D) 9 and 2    (E) 3 and 8