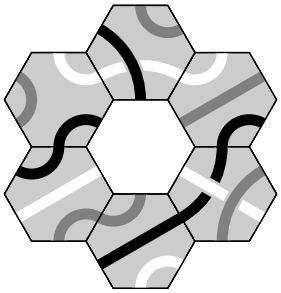
Year 11 to 13 (ENGLISH VERSION)

Thursday, 17th March 2016 Time allowed: 75 minutes

1. For each question exactly one of the 5 options is correct.
2. Each participant is given 30 points at the beginning. For each correct answer 3, 4 or 5 points are added. No answer means 0 points are added. If a wrong answer is given, one quarter of the points is subtracted, i. e. 0.75 points, 1 point or 1.25 points, respectively. At the end, the maximum number of points is 150, the minimum is 0.
3. Calculators are not allowed.

3 point problems

A1 Which of the following tiles fits in the middle of the puzzle such that black lines meet black lines, grey lines meet grey lines and white lines meet white lines?



(**A**)  (**B**)  (**C**)  (**D**)  (**E**) 

A2 $\frac{1}{10}+\frac{1}{100}+\frac{1}{1000}=$

(**A**) $\frac{3}{1110}$ (**B**) $\frac{3}{100}$ (**C**) $\frac{111}{1000}$ (**D**) $\frac{3}{111}$ (**E**) $\frac{111}{1110}$

A3 Oleg has a chocolate bar that has 8 rows of 6 pieces of chocolate each. Oleg eats all the edge pieces. He then gives all the edge pieces of the remaining bar to his girlfriend. How many pieces of chocolate does Oleg have left?

(**A**) 8 (**B**) 12 (**C**) 16 (**D**) 18 (**E**) 24

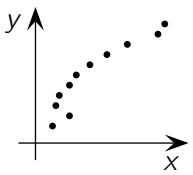
A4 Annika rolls a die whose faces are labelled with the numbers 1, −2, 3, −4, 5, −6. What is the probability that Annika rolls a number less than 3?

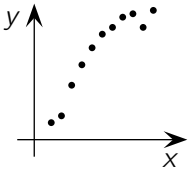
(**A**) $\frac{1}{6}$ (**B**) $\frac{1}{3}$ (**C**) $\frac{1}{2}$ (**D**) $\frac{2}{3}$ (**E**) $\frac{5}{6}$

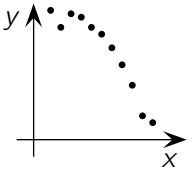
A5 The sum of the ages of Tom and John is 23, the sum of the ages of John and Alex is 24 and the sum of the ages of Tom and Alex is 25. What is the age of the oldest one?

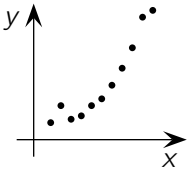
(**A**) 12 (**B**) 13 (**C**) 14 (**D**) 15 (**E**) 16

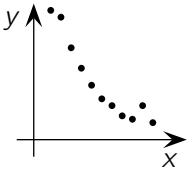
A6 Felix entered the measured data of an experiment in a coordinate system, as shown on the right, but accidentally swapped the values for x and y. Which of the following shows the correct diagram?

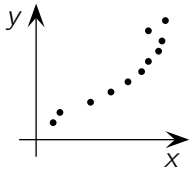


(**A**) 

(**B**) 

(**C**) 

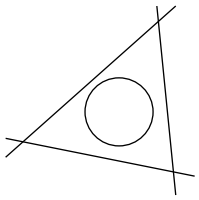
(**D**) 

(**E**) 

A7 How many integers are greater than 2015×2017 but less than 2016×2016?

(**A**) 0 (**B**) 1 (**C**) 2015 (**D**) 2016 (**E**) 2017

A8 To enclose a circle in the plane 3 straight lines are needed, as shown on the right. What is the smallest number of planes that are needed to enclose a sphere in three-dimensional space?



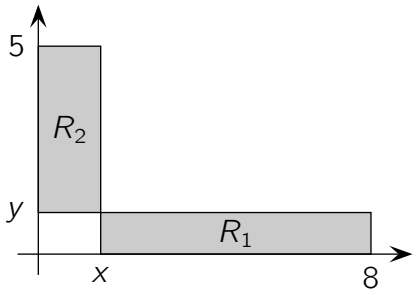
(**A**) 3 (**B**) 4 (**C**) 5 (**D**) 6 (**E**) 7

A9 Kim, Audrey and Hanna are riding bicycles. Kim starts behind Audrey and Audrey starts behind Hanna. During the ride, each of the girls is passed exactly once by exactly one of the other girls. In how many different orders can they finish the ride?

(**A**) only one (**B**) two (**C**) three (**D**) four (**E**) six

A10 The two grey rectangles R_1 and R_2 in the figure on the right have the same area (*diagram not to scale*).

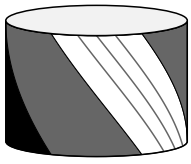
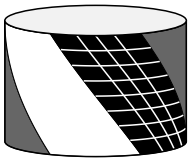
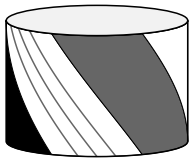
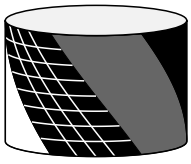
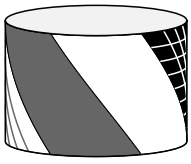
What is the value of $\frac{x}{y}$?



(**A**) 1 (**B**) $\frac{3}{2}$ (**C**) $\frac{4}{3}$ (**D**) $\frac{7}{4}$ (**E**) $\frac{8}{5}$

4 point problems

B1 Ran has decorated her drum for a festival with six wide stripes. Only four of the following pictures show Ran's drum. Which one does not show Ran's drum?

(**A**)  (**B**)  (**C**)  (**D**)  (**E**) 

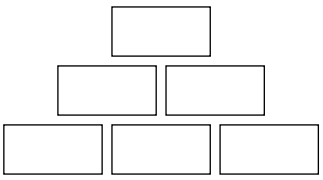
B2 Football fans were travelling to a match in 16 minibuses. There was an equal number of people in each of them. On the way 4 minibuses broke down and the fans from these buses got on the remaining ones. After that there were 5 more fans in each of these minibuses. How many fans were travelling to the match?

(**A**) 80 (**B**) 128 (**C**) 192 (**D**) 225 (**E**) 240

B3 If $x^2 - 8x + 16 = 0$, then $x + \frac{16}{x}$ equals

(**A**) -8 (**B**) -4 (**C**) 0 (**D**) 4 (**E**) 8

B4 Luca wants to put a positive integer greater than 1 in each rectangle of the pyramid shown on the right. The integers written in the middle and the top layers of the pyramid must be the product of the two numbers directly underneath. Which of the following numbers *cannot* appear in the top rectangle?



(**A**) 56 (**B**) 84 (**C**) 90 (**D**) 105 (**E**) 220

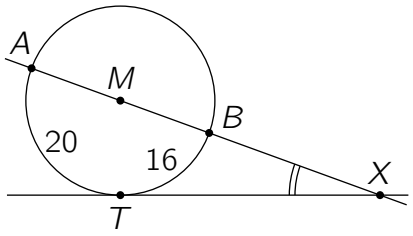
B5 Four positive integers a, b, c, d are such that $a+2 = b-2 = c\times 2 = d:2$. Which is the largest of these four integers?

(**A**) a (**B**) b (**C**) c (**D**) d (**E**) This is not uniquely determined.

B6 Harry's watch is 3 minutes fast, but he believes it is 5 minutes slow. Sally's watch is 5 minutes slow, but she believes it is 3 minutes fast. At the same moment, each of them looks at their own watch. Harry thinks it is 12:00. What time does Sally think it is?

(**A**) 11:44 (**B**) 11:56 (**C**) 12:00 (**D**) 12:04 (**E**) 12:16

B7 Let k be a circle, AB a straight line through its centre M and X the point of intersection of AB with the tangent in T. Arc $\overset{\frown}{\mathrm{AT}}$ has length 20 cm and arc $\overset{\frown}{\mathrm{TB}}$ has length 16 cm (*diagram not to scale*). What is the size of angle $\measuredangle AXT$?



(**A**) 30° (**B**) 24° (**C**) 18° (**D**) 15° (**E**) 10°

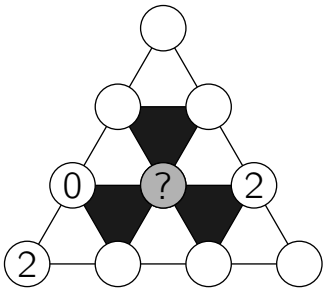
B8 On the Island of Knights and Knaves every citizen is either a Knight (who always speaks the truth) or a Knave (who always lies). One day on the island 7 people were sitting in a circle around a bonfire. Each of these 7 citizens claimed: "I'm sitting between two Knaves!" How many Knaves were sitting around the bonfire?

(**A**) 3 (**B**) 4 (**C**) 5
(**D**) 6 (**E**) More information is needed.

B9 In rectangle $ABCD$ the length of the side $\overline{BC}$ is half the length of the diagonal $\overline{AC}$. Let M be a point on $\overline{CD}$ such that $|\overline{AM}| = |\overline{MC}|$. What is the size of angle $\measuredangle CAM$?

(**A**) 15° (**B**) 22.5° (**C**) 27.5° (**D**) 30° (**E**) 36°

B10 In each of the 10 circles in the figure either 0 or 1 or 2 is written. The sum of the numbers in the vertices of any white triangle is divisible by 3, while the sum of the numbers in the vertices of any black triangle is *not* divisible by 3. What numbers can be written in the grey circle?



(**A**) only 0 (**B**) only 1 (**C**) only 2
(**D**) either 0 or 1 (**E**) All three numbers are possible.

5 point problems

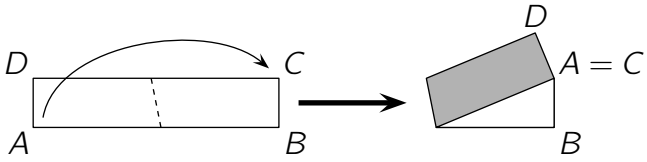
C1 It takes 4 hours for a motorboat to travel downstream from Sourceton to Mouthville. To return upstream from Mouthville to Sourceton it takes the motorboat 6 hours. How many hours would it take a wooden log to be carried from Sourceton to Mouthville by the current, assuming it is unhindered by any obstacles?

(**A**) 10 (**B**) 24 (**C**) 12 (**D**) 22 (**E**) 16

C2 How many different real solutions does the equation $(x^2-4x+5)^{(x^2+x-30)} = 1$ have?

(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) infinitely many

C3 The rectangular strip of paper shown is 5 cm wide and 25 cm long. It is white on one side and grey on the other. The strip is folded as indicated such that vertex A coincides with vertex C. What is the area of the visible white part of the folded strip in the diagram?

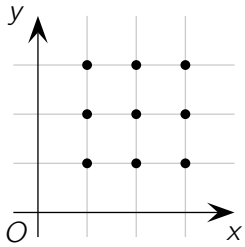


(**A**) 25 cm^2 (**B**) 27.25 cm^2 (**C**) 27.5 cm^2 (**D**) 30 cm^2 (**E**) 31.25 cm^2

C4 Three three-digit numbers are formed using the nine digits $1, 2, \ldots, 9$ such that each digit is used exactly once. Which of the following numbers could not be equal to the sum of these three numbers?

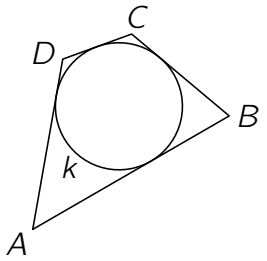
(**A**) 1500 (**B**) 1503 (**C**) 1512 (**D**) 1521 (**E**) 1575

C5 How many quadratic functions $f(x) = ax^2 + bx + c$ with $a \neq 0$ have a graph passing through at least three of the nine marked points?



(**A**) 6 (**B**) 15 (**C**) 20 (**D**) 22 (**E**) 27

C6 A quadrilateral $ABCD$ contains an inscribed circle k (i. e. a circle tangent to the four sides of the quadrilateral). The ratio of the perimeter of the quadrilateral $ABCD$ to the perimeter of the circle k is 4 : 3 (*diagram not to scale*). What is the ratio of the area of the quadrilateral $ABCD$ to the area of the circle k?



(**A**) $4 : \pi$ (**B**) $3\sqrt{2} : \pi$ (**C**) 16 : 9 (**D**) $\pi : 3$ (**E**) 4 : 3

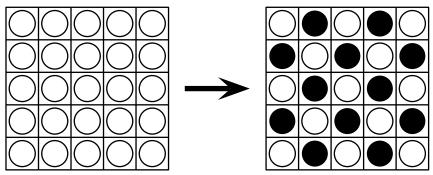
C7 A cube is dissected into six pyramids by connecting a point in the interior of the cube with each vertex of the cube. The volumes of five of these pyramids are 2, 5, 10, 11 and 14. What is the volume of the sixth pyramid?

(**A**) 4 (**B**) 6 (**C**) 9 (**D**) 12 (**E**) 22

C8 The positive integer N has exactly six distinct positive divisors (including 1 and N). The product of five of these divisors is 648. Which of the following is the sixth divisor of N?

(**A**) 4 (**B**) 8 (**C**) 9 (**D**) 12 (**E**) 24

C9 In each cell of a 5×5 square there is a token that is black on one side and white on the other. Initially all tokens are placed with the white side facing up. On each move three tokens in consecutive cells in a row or in a column are flipped over. What is the smallest number of moves needed in order to obtain the chessboard colouring shown on the right?



(**A**) 6 (**B**) 8 (**C**) 10 (**D**) 11 (**E**) 12

C10 Alexandra chose a positive integer n and wrote down the sum of all positive integers from 1 to n. The prime number p divides this sum, but not any of the summands. Which of the following could be the value of $n + p$?

(**A**) 217 (**B**) 221 (**C**) 229 (**D**) 245 (**E**) 269